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TECHNICAL NOTE

Surface temperature of a hot film on a wall in shear flow

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INTRODUCTION

THE HEAT transfer from a hot film on a wall into a moving fluid has been a topic investigated by many researchers because the hot-film technique is widely used to measure skin friction. Leveque [1] first gave a similarity solution for a semi-infinite hot film in an homogenous shear flow. Further theoretical works were made by Fage and Falkner [2], Lighthill [3], and Liepmann [4]. More recent analytical and numerical studies are included in references [5–12]. Experimental studies have also been made to determine the calibration relation between heat transfer from a hot film into the fluid and local shear stress [13, 14]. Nevertheless, the surface temperature field near the hot film has never been measured. This paper contains both a theoretical analysis and experimental measurement of the surface temperature on a hot film. First, solving a Volterra integral equation with a strongly singular kernel, we derive strikingly simple expressions for surface temperature on a uniform temperature film and a hot film with an arbitrary heat source distribution on an adiabatic wall at large Peclet number. Then, using a newly developed fluorescent paint technique with high spatial resolution [15–20], we measure the surface temperature distribution for a commercially available hot-film sensor and a heated copper strip on Kapton film. These measurements contribute useful background data for design improvement of hot-film sensors. The theoretical solutions give a greater insight to our understanding of the measured temperature distributions.

SIMPLE ANALYTICAL SOLUTIONS

In this section, exact analytical solutions for a hot film on an adiabatic wall at large Peclet number are derived. The significance of these solutions is that they provide a standard reference for experimental and numerical results. Figure 1 shows a two-dimensional hot film on an adiabatic wall in an homogeneous shear flow. It is assumed that the thickness of the hot film is so small that the film does not disturb the flow.

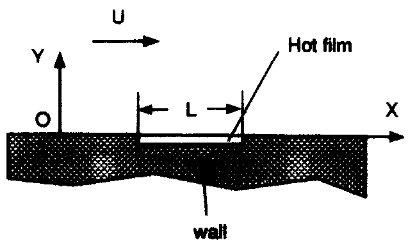

FIG. 1. Hot film on a wall and coordinate system.

The heat transfer from a hot film into a moving fluid is described by the two-dimensional energy equation

$$u(Y)\frac{\partial T}{\partial X} = c_d\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) \quad (1)$$

where T is the temperature and c_d is the thermal diffusivity coefficient of the fluid. For an homogeneous shear flow, $u(Y) = SY$, where S is the shear rate. We introduce the non-dimensional variables

$$\theta(x,y) = \frac{T(x,y) - T_\infty}{T_m - T_\infty} \quad (2)$$

$$x = X/L \quad y = Y(Pe)^{1/3}/L \quad (3)$$

$$Pe = SL^2/c_d \quad L = X_T - X_L \quad (4)$$

where Pe is the Peclet number, T_∞ the free-stream temperature and T_m a reference temperature. X_L and X_T denote the positions of the leading and trailing edges of the hot film respectively and L is the streamwise length of the hot film. Thus, (1) becomes

$$y\theta_x = \theta_{xx}(Pe)^{-2/3} + \theta_{yy}. \quad (5)$$

When $Pe \to \infty$, (5) becomes

$$y\theta_x = \theta_{yy}. \quad (6)$$

On the boundaries,

$$\theta(x,0) = W(x), \quad \theta(x,\infty) = 0 \quad (7)$$

where $W(x)$ is the non-dimensional wall temperature to be determined later by the heat flux condition on the adiabatic wall. Also, for a film on an adiabatic wall, we can assign the origin of the coordinate system far upstream of the hot film so that the following condition is satisfied:

$$\theta(0,y) = 0. \quad (8)$$

Applying the Laplace transform and convolution theorem, we obtain a solution to (6) with (7) and (8):

$$\theta(x,y) = \frac{y}{3^{2/3}\Gamma(1/3)}\int_0^x (x-t)^{-4/3}W(t)\exp\left(-\frac{y^3}{9(x-t)}\right)dt. \quad (9)$$

At this stage, (9) is a formal solution because $W(x)$ remains unknown. When $W(x)$ is taken to be a step function, (9) is naturally reduced to Leveque's well-known similarity solution for a heated semi-infinite plate [1].

Uniform temperature film

For a uniform-temperature film (UT film), the non-dimensional wall temperature distribution has the following form:

NOMENCLATURE

c_d	thermal diffusivity coefficient of fluid
E	activation energy
I	fluorescent intensity
L	streamwise length of hot film
Pe	Peclet number, SL^2/c_d
Q	non-dimensional heat source strength constant
R	universal gas constant
S	shear rate
T	temperature
T_∞	free-stream temperature
T_m	reference temperature
T_s	surface temperature
$W(x)$	non-dimensional surface temperature, $(T_s-T_\infty)/(T_m-T_\infty)$
w	half-span width of hot films
X, Y	streamwise and vertical coordinates, respectively
x, y	X/L and $Y(Pe)^{1/3}/L$, respectively
X_L, X_T	positions of hot film leading and trailing edges, respectively
x_L, x_T	X_L/L and X_T/L, respectively
x_1, x_2	$x-x_T$ and $x-x_L$, respectively
Z	spanwise coordinate.

Greek symbol

θ	non-dimensional temperature, $(T-T_\infty)/(T_m-T_\infty)$.

$$W(x) = \frac{T_s(x)-T_\infty}{T_m-T_\infty} = H(x-x_L) - H(x-x_T) + H(x-x_T)W_1(x-x_T) \quad (10)$$

where $T_s(x)$ denotes the surface temperature, $W_1(x-x_T)$ is the unknown non-dimensional wall temperature in the thermal wake behind the hot film and $H(x)$ is the Heaviside function ($H(x) = 1$ when $x \geqslant 0$, $H(x) = 0$ when $x < 0$). It is noted that T_m in (10) is taken as the maximum temperature on the hot film. For an adiabatic wall, the heat-flux condition is

$$\theta_y(x, 0) = 0. \quad (11)$$

Substitution of (9) and (10) into (11) yields a Volterra integral equation of the first kind with a strongly singular kernel for $W_1(x)$:

$$3[x_1^{-1/3}-(x_1+1)^{-1/3}] + \int_0^{x_1} \frac{W_1(t)}{(x_1-t)^{4/3}}\,dt = 0 \quad (12)$$

where $x_1 = x-x_T = (X-X_T)/L$. The suitable boundary condition for (12) is that $W_1(0) = 1$. The strongly singular integral in (12) is meaningful only in the sense of 'the finite part' of an improper integral, which was first introduced by Hadamard [21]. Following Hadamard's procedure, 'the finite part' (F.P.) of the integral in (12) is evaluated by

$$\text{F.P.}\int_0^{x_1} \frac{W_1(t)}{(x_1-t)^{4/3}}\,dt = \lim_{\varepsilon\to 0}\left[\int_0^{x_1-\varepsilon} \frac{W_1(t)}{(x_1-t)^{4/3}}\,dt + G(\varepsilon)\right] \quad (13)$$

where ε is a small positive real number and $G(\varepsilon)$ is a function of ε which cancels the divergent terms in the integral out exactly. Integration by parts and elimination of the divergent term by choosing $G(\varepsilon) = -3\varepsilon^{-1/3}W_1(x_1-\varepsilon)$ lead to

$$\text{F.P.}\int_0^{x_1} \frac{W_1(t)}{(x_1-t)^{4/3}}\,dt = -3x_1^{-1/3} - 3\int_0^{x_1} \frac{W_1'(t)}{(x_1-t)^{1/3}}\,dt \quad (14)$$

where the prime denotes differentiation with respect to t. Replacing the singular integral in (12) by its finite part, (14) yields an equivalent integral equation with a weakly singular kernel:

$$(x_1+1)^{-1/3} + \int_0^{x_1} \frac{W_1'(t)}{(x_1-t)^{1/3}}\,dt = 0. \quad (15)$$

Use of the Laplace transform and convolution theorem leads to a simple solution for the surface temperature behind a UT film $W_1(x_1)$:

$$W_1(x_1) = 1 - \frac{\sqrt{3}}{2\pi}\int_0^{x_1}(1+t)^{-1}t^{-2/3}\,dt. \quad (16)$$

Expression (16) is in excellent agreement with Ling's numerical solution [5] to the energy equation for a UT film at $Pe = 5000$, as shown in Fig. 2. When x_1 is large, the asymptotic expression for $W_1(x_1)$ is

$$W_1(x_1) \to \frac{3^{3/2}}{4\pi}x_1^{-2/3} \approx 0.413497x_1^{-2/3}. \quad (17)$$

This recovers Ling's similarity solution [5] for the far thermal wake. Based on numerical results, Ling gave a coefficient of 0.413. Here, (17) provides an exact value of $3^{3/2}/4\pi$.

Hot film with an arbitrary heat source distribution

For a film with an arbitrary heat source distribution, the non-dimensional heat flux condition on the adiabatic wall is

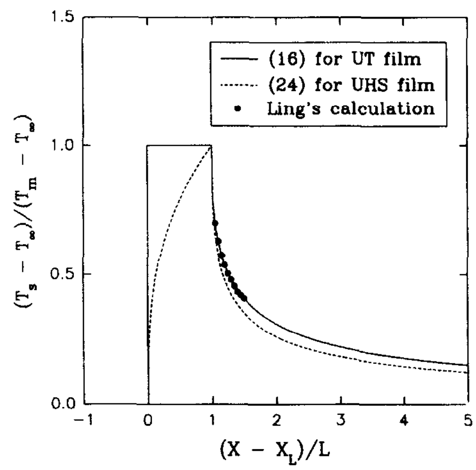

FIG. 2. Comparison of the analytic expressions of surface temperature for the UT and UHS films with Ling's numerical calculation [5].

$$-\left.\frac{\partial\theta}{\partial y}\right|_{y=0} = QF(x-x_{\rm L})\,[H(x-x_{\rm L})-H(x-x_{\rm T})] \quad (18)$$

where Q is a dimensionless heat-source strength constant and $F(x-x_{\rm L})$ is a non-dimensional shape function that specifies the heat-source distribution on the film and varies between 0 and 1. On the adiabatic wall, $W(x)$ has the form

$$W(x) = \frac{T_{\rm s}(x)-T_\infty}{T_{\rm m}-T_\infty} = H(x-x_{\rm L})\,W_2(x-x_{\rm L}). \quad (19)$$

Substitution of (9) and (19) into (18) yields a singular integral equation for $W_2(x_2)$:

$$QF(x_2)\,[H(x_2)-H(x_2-1)] + \frac{1}{3^{2/3}\Gamma(1/3)}\int_0^{x_2}\frac{W_2(t)}{(x_2-t)^{4/3}}\,{\rm d}t = 0 \quad (20)$$

where $x_2 = x-x_{\rm L} = (X-X_{\rm L})/L$. The boundary condition for (20) is that $W_2(0) = 0$. As shown above, the finite part of the singular integral in (20) can be evaluated under the condition $W_2(0) = 0$. The solution of (20) is

$$W_2(x_2) = \frac{Q}{3^{1/3}\Gamma(2/3)}\left[H(x_2)\int_0^{x_2}\frac{F(t)\,{\rm d}t}{(x_2-t)^{2/3}} - H(x_2-1)\int_1^{x_2}\frac{F(t)\,{\rm d}t}{(x_2-t)^{2/3}}\right]. \quad (21)$$

When $x_2 \to \infty$,

$$W_2(x_2) \to \frac{Q}{3^{1/3}\Gamma(2/3)}\,x_2^{-2/3}\int_0^1 F(t)\,{\rm d}t. \quad (22)$$

The asymptotic expression (22) indicates that the surface temperature in the far thermal wake always obeys the $-2/3$ power-law no matter what the heat-source distribution on the hot film is. In particular, for a uniform-heat-source film (UHS film) on which $F(x_2) = 1$, (21) is reduced to a closed-form solution

$$W_2(x_2) = \frac{3^{2/3}Q}{\Gamma(2/3)}\,[x_2^{1/3}H(x_2)-(x_2-1)^{1/3}H(x_2-1)]. \quad (23)$$

If $T_{\rm m}$ is taken as the maximum surface temperature, without loss of generality, $W_2(x_2)$ is simply rewritten as

$$W_2(x_2) = x_2^{1/3}H(x_2)-(x_2-1)^{1/3}H(x_2-1). \quad (24)$$

The behavior of $W_2(x_2)$ is shown in Fig. 2.

EXPERIMENTS

Surface temperature distributions on a commercial hot-film sensor (TSI 1237) and a heated copper strip on Kapton film were measured using the fluorescent paint technique. The comparison of the experimental temperature distributions with the simple analytical solutions obtained above sheds light on the heat transfer characteristics of hot films.

Fluorescent paint technique

A temperature-sensitive fluorescent paint technique for remote surface temperature mapping has been developed [15, 16] and used in fluid mechanics experiments [17–20]. Figure 3 is a schematic illustrating this technique. Under ultraviolet excitation, the paint radiates light of longer wavelength through the photophysical process of fluorescence. Due to thermal quenching, the fluorescent emission intensity decreases with increasing temperature. The relationship between emission intensity and temperature is determined experimentally using a calibration set-up [20]. An Arrhenius relation can be used to express the temperature dependence of the paint's fluorescent intensity:

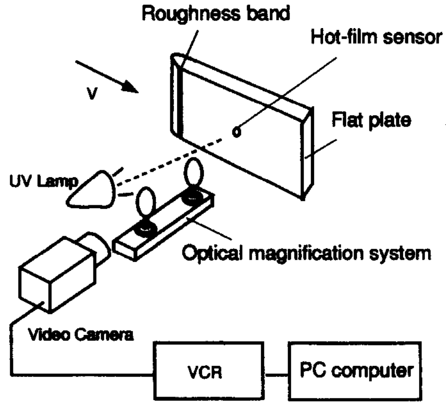

FIG. 3. Experimental set-up for surface-temperature mapping of a hot film in a turbulent boundary layer.

$$\ln\left(\frac{I(T)}{I(T_{\rm r})}\right) = \frac{E}{R}\left(\frac{1}{T}-\frac{1}{T_{\rm r}}\right) \quad (25)$$

where I is the fluorescent intensity, $T_{\rm r}$ a reference temperature, E the activation energy and R the universal gas constant. The coefficient E/R is evaluated from the experimental calibration [20]. The fluorescent paint used in this test is a combination of the rare-earth chelate europium thenoyltrifluoroacetonate (EuTTA) and model airplane dope (a polymer solution). The coefficient E/R for this paint is 2955 between 285 and 323 K. The fluorescent paint (thickness of about 0.01 mm) is applied to the hot-film sensor on a flat plate. A video camera views the hot film and surrounding area through an optical magnification system and the images are recorded on a VCR. A personal computer equipped with an 8-bit (256 grey levels) frame grabber board digitizes the images at a 512 by 512 pixel spatial resolution. Relation (25) is used to determine the temperature at each pixel location in the images.

Commercial hot-film sensor

The commercial sensor (the TSI 1237) is mounted flush with the surface of a flat plate with a 1:6 elliptical nose installed in the low speed blow-down wind tunnel at Purdue (Fig. 3). The sensor is located 0.37 m downstream from the nose of the plate. A roughness band near the plate leading edge is used to produce artificial flow transition so that the boundary-layer is fully developed turbulent. Figure 4 shows the dimensions of the TSI 1237 hot-film sensor. It has a 0.127 mm streamwise length and a spanwise width of 1 mm. The cold resistance of the sensor is 5.14 Ω. An image of the EuTTA paint on the sensor operating at a low overheat ratio of 1.07 is shown in Fig. 5. The dark region (low fluorescent emission) corresponds to high temperature. Flow moves from right to left in Fig. 5 with a freestream velocity of 26 m s^{-1}. The non-dimensional streamwise surface temperature distributions at three spanwise locations are plotted in Fig. 6 along with the analytical solutions for the UT and UHS films on an adiabatic wall, where Z is the spanwise coordinate, L is the streamwise length, w is the half-span width, and $T_{\rm m}$ is the maximum surface temperature. Notice that the measured temperature distributions are essentially the same at the three spanwise locations. The temperature plots for the TSI film appear to be near-symmetric and deviate largely from the theoretical distributions with an adiabatic wall. This deviation is due to both heat conduction to the substrate and a streamwise diffusion effect (the finite Peclet number effect) which are neglected in the simple analytical solutions.

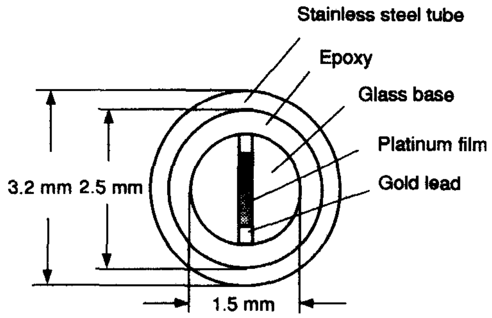

FIG. 4. Configuration of the TSI 1237 hot-film sensor.

Furthermore, since the streamwise diffusion effect is limited in the neighborhood of the leading and trailing edges [5, 7, 8], it is inferred that the deviation is mainly attributed to heat conduction to the glass substrate.

The measured spanwise temperature distribution on the TSI hot film is shown in Fig. 7. In order to analyze the spanwise temperature distribution of the hot film, a simple lumped model can be formulated based on an energy balance among the heat generation, convective heat loss to the fluid and heat conduction into the substrate. If the hot film has a very large aspect ratio such that the heat conduction into the substrate along the spanwise direction can be neglected, the spanwise temperature of the hot film can be described by an ordinary differential equation similar to that for a cylindrical hot wire given by Davies and Fisher [22]. For $\Delta T = T_s - T_\infty$, the equation is

$$\frac{d^2\Delta T}{dZ^2} + K_1 \Delta T = K_2 \tag{26}$$

with the boundary condition $\Delta T(Z = \pm w) = C$ and $d(\Delta T)/dZ(Z = 0) = 0$, where K_1 and K_2 are parameters including the effects of the heat generation, convective heat transfer to the fluid, heat conduction into the substrate and the geometric scales of the hot film. In contrast to the case

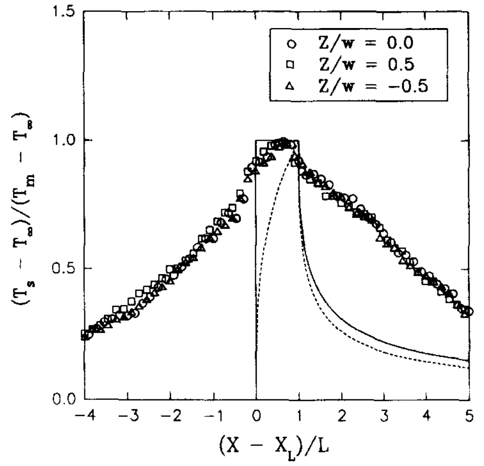

FIG. 6. Streamwise surface temperature of the TSI 1237 hot-film sensor operating at overheat ratio 1.07 in shear flow.

of the hot wire, the constant C is not zero for a hot film. The solution to (26) is

$$T_s - T_\infty = \frac{K_2}{K_1}\left(\left(\frac{CK_1}{K_2} - 1\right)\frac{\cosh(|K_1|^{1/2}Z)}{\cosh(|K_1|^{1/2}w)} + 1\right). \tag{27}$$

As shown in Fig. 7, the theoretical distribution (27) fits the experimental data well when $|Z|/w < 1.2$. Outside of this region, equation (27) underestimates the temperature. This indicates that the heat conduction into the substrate along the spanwise direction is no longer negligible.

Copper strip on Kapton film

For further comparison with the simple analytical solutions, we measure the surface temperature distribution of a heated copper strip on Kapton film ($k = 0.12$ W m^{-1} K^{-1}) which insulates about 10 times better than glass ($k = 1.1$ W m^{-1} K^{-1}). The copper strip has a 0.8 mm streamwise length,

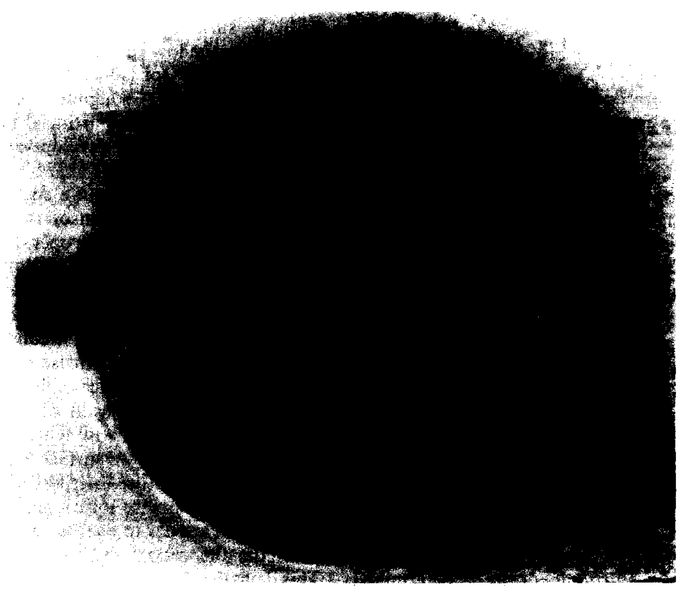

FIG. 5. Fluorescent image of the TSI 1237 hot-film sensor operating at overheat ratio 1.07 in shear flow. The arrow indicates flow direction.

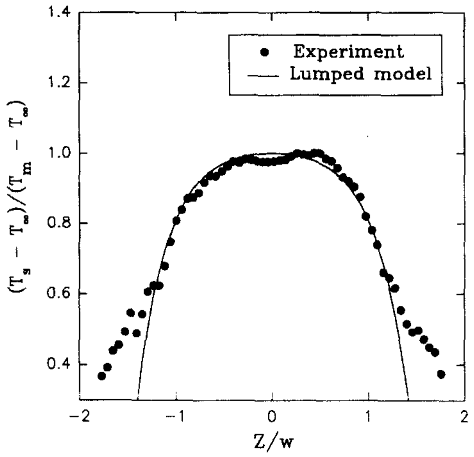

FIG. 7. Spanwise surface temperature of the TSI 1237 hot-film sensor operating at overheat ratio 1.07 in shear flow.

a spanwise width of 24 mm and a 0.008 mm thickness. Its cold resistance is 0.9 Ω. The copper strip is heated electrically by supplying a 0.1 A current through it. The measured streamwise surface temperature of the heated copper strip is shown in Fig. 8. The measured surface temperature in the far thermal wake asymptotically follows the $-2/3$ power-law predicted by the analytical solutions for a hot film on an adiabatic wall. On the heated strip, however, the departure of the measured temperature distribution from the theoretical distributions indicates that the strip is neither a UT nor UHS film. This deviation is due to heat conduction to the substrate and a non-uniform heat-source distribution.

CONCLUSIONS

Exact analytical solutions for surface temperature are obtained for a two-dimensional uniform-temperature (UT) hot film and a hot film with an arbitrary heat-source distribution on an adiabatic wall at large Peclet number. These solutions show that the surface temperature in the far thermal wake behind the hot film always asymptotically follows the $-2/3$ power-law. Nevertheless, the measured streamwise surface temperature distribution on a TSI 1237 hot film sensor deviates largely from the simple analytical solutions and appears to have a near-symmetric distribution. This fact reveals that the heat conduction from the TSI sensor to the glass substrate is dominant over convection to the fluid. The measured spanwise temperature on the TSI sensor is in reasonable agreement with a simple lumped model. For the heated copper strip on an insulating Kapton film, the surface temperature in the far thermal wake approaches the $-2/3$ power-law as predicted by the analytical solutions. However, on the strip, the temperature distribution still differs from the solutions for UT and UHS hot films. The results in this paper show that the commercially available hot film is neither a UT nor UHS film and heat conduction into the substrate is of basic importance in hot-film heat transfer.

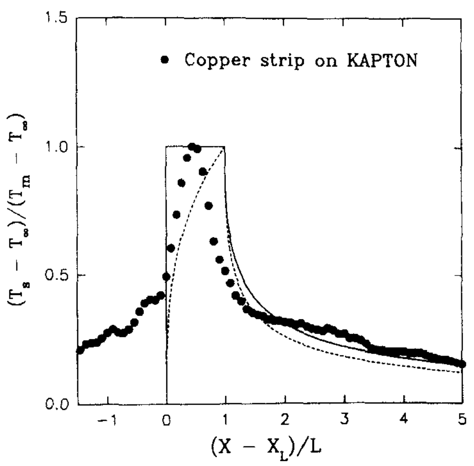

FIG. 8. Streamwise surface temperature of the heated copper strip on Kapton film in shear flow.

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